

Code: 20BS1101

I B.Tech - I Semester – Regular Examinations – JULY 2021**CALCULUS AND LINEAR ALGEBRA**
(Common for CIVIL, EEE, ME, ECE, CSE, IT)

Duration: 3 hours

Max. Marks: 70

Note: 1. This paper contains questions from 5 units of Syllabus. Each unit carries 14 marks and have an internal choice of Questions.
2. All parts of Question must be answered in one place.

UNIT – I

1. a) Find the rank of the matrix $A = \begin{bmatrix} 1 & 3 & 4 & 5 \\ 1 & 2 & 6 & 7 \\ 1 & 5 & 0 & 10 \end{bmatrix}$ by reducing into Normal form. 7 M
- b) Show that the equations $3x + 4y + 5z = a$,
 $4x + 5y + 6z = b$, $5x + 6y + 7z = c$ does not have a solution unless $a + c = 2b$. 7 M
- OR
2. a) Reduce the matrix $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 5 & 4 \end{bmatrix}$ into Echelon form and hence find its rank. 7 M
- b) Solve the following system of linear equations
 $x + 3y + 2z = 0$, $2x - y + 3z = 0$, $3x - 5y + 4z = 0$, $x + 17y + 4z = 0$ 7 M

UNIT – II

3. Find the Eigen values and Eigen Vectors of

$$A = \begin{bmatrix} 2 & 3 & -2 \\ -2 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}.$$

14 M

OR

4. a) Verify the Cayley- Hamilton theorem for the matrix

$$A = \begin{bmatrix} 1 & \sqrt{2} & 0 \\ \sqrt{2} & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and find its inverse by using this}$$

7 M

theorem.

b) Prove that zero is the eigen value of a matrix if and only if the matrix is singular.

7 M

UNIT-III

5. a) Verify the Rolle's theorem for the function

$$f(x) = (x-a)^m(x-b)^n \text{ in } [a,b]$$

7 M

b) Apply mean value theorem, show that

$$\frac{\pi}{4} + \frac{3}{25} < \tan^{-1} \frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}.$$

7 M

OR

6. a) Verify Cauchy's Mean value theorem for $f(x) = \frac{x^3}{4} - 4x$ and $g(x) = x^2$ in $[0,3]$.

7 M

b) Obtain the Maclaurin's series expansion of $f(x) = \cos x$.

7 M

UNIT – IV

7. a) If $u = x + y + z$, $uv = y + z$, $uvw = z$ then, prove that

$$\frac{\partial(x, y, z)}{\partial(u, v, w)} = u^2 v. \quad 7 \text{ M}$$

b) Examine for minimum and maximum values of $\sin x + \sin y + \sin(x + y)$. 7 M

OR

8. a) Find the extreme values of $x^3 + y^3 - 3axy$, where $a > 0$. 7 M

b) Show that rectangular solid of maximum volume that can be inscribed in a given sphere is cube. 7 M

UNIT – V

9. a) Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) dy dx$. 7 M

b) Evaluate $\int_0^1 \int_0^{\sqrt{x}} \int_0^{\sqrt{xy}} xyz dz dy dx$ 7 M

OR

10. a) Evaluate $\int_{-a}^a \int_{-b}^b \int_{-c}^c (x^2 + y^2 + z^2) dx dy dz$. 7 M

b) By using double integrals find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$. 7 M